Improvement of Convergence in Time-Harmonic Eddy Current Analysis by Hierarchical Domain Decomposition Method

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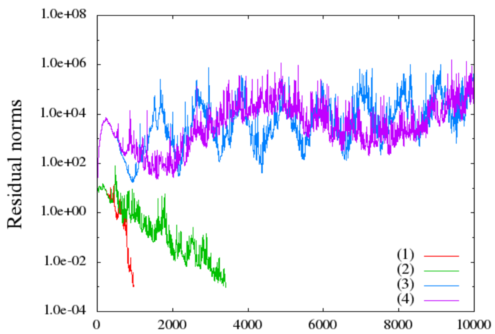
This paper deals with a large-scale 3D time-harmonic eddy current analysis using the Hierarchical Domain Decomposition Method (HDDM). To improve the convergence of the interface problem of the HDDM and to reduce the computation time, the HDDM is applied to the mixed formulation with the Lagrange multiplier. Because the conventional formulation is singular, it is expected that the characteristic of convergence of the interface problem is made bad. Therefore, in this paper, the mixed formulation of the *A* method with the Lagrange multiplier that is not singular is considered. As a result, the convergence of the interface problem is much improved, and the time-harmonic eddy current problem with 3.5 billion degrees of freedom is solved in about 9 hours.

***Index Terms*—Finite Element Method, Eddy Current Problem, Domain Decomposition Method, Parallel Processing.**

I. Introduction

A computational object tends to be large and complicated for numerical analysis recently. In addition, subdivision of the mesh is performed for the improvement of accuracy. Therefore, large-scale computations are increasingly important in electromagnetic field problems. To meet this requirement, we have already introduced Hierarchical Domain Decomposition Method (HDDM) [1]-[4] to 3D time-harmonic eddy current problems using the A method and the A-ϕ method with the continuity of the electric current density[5]. To confirm effectiveness of our method, the model for the accuracy verification of the eddy current analysis that uses the solenoidal coil with unlimited length was analyzed changing its Degrees of Freedom (DOF) several times up to 44 million DOF. The computations were performed with a PC cluster that consists of 32 PCs. As a result, a time-harmonic eddy current problem with 44 million DOF was successfully solved in about 4.8 hours [6].



(1) 44M DOF (A-ϕ method), (2) 44M DOF (A method)
(3) 55M DOF (A-ϕ method), (4) 55M DOF (A method)
Fig. 1 Convergence histories by the previous method

The possibility of large-scale analysis in 3D time-harmonic eddy current problems that are represented by complex numbers has been shown. However, a computation of the same model with 55 million DOF diverged (Fig. 1). Therefore we have to improve convergence of the interface problem. In this paper, to improve the convergence of the interface problem of the HDDM and to reduce the computation time, the HDDM is applied to the mixed formulation with the Lagrange multiplier that is not singular in the A method of the time-harmonic eddy current problem.

II. Methods

In the A method of the time-harmonic eddy current problem, the following finite element formulation [7] is considered generally.

$$(\nu \operatorname{rot} A_h, \operatorname{rot} A_h^*) - (i\omega\sigma A_h, A_h^*) = (\tilde{J}_h,\ A_h^*), \qquad (1)$$

where ν denotes the magnetic reluctivity [m/H], ω the angular frequency [rad/s], σ the conductivity [S/m], J the excitation current density [A/m^2], and i the imaginary unit. A that denotes the magnetic vector potential is used as an unknown complex function. Because the equation (1) is singular, it is expected that the characteristic of convergence of the interface problem is made bad.

In this paper, we consider the mixed formulation with the Lagrange multiplier p [8].

$$(\nu \operatorname{rot} A_h, \operatorname{rot} A_h^*) - (i\omega\sigma A_h, A_h^*) + (\operatorname{grad} p_h, A_h^*) = (\tilde{J}_h,\ A_h^*), \qquad (2a)$$

$$(A_h, \operatorname{grad} p_h^*) = 0. \qquad (2b)$$

The HDDM is introduced to the equation (2).

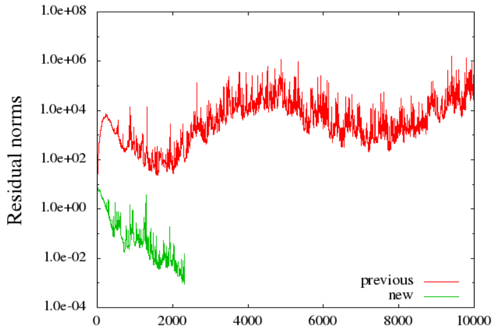
III. Numerical Results

A. Numerical model

We consider the model for the accuracy verification of the eddy current analysis that uses the solenoidal coil with unlimited length [5]. TABLE I shows numbers of elements, nodes and subdomains. A simplified block diagonal scaling is used as the preconditioner and the convergence criterion is 1.0e-03 in the interface problem. In the previous method, ICCOCG method is used as the subdomain solver with the accelerative parameter 1.2 and its convergence criterion is 1.0e-09. In the new method, the *LU* decomposition with the pivoting is used as the subdomain solver.

TABLE I
NUMBER OF ELEMENTS, NODES AND SUBDOMAINS

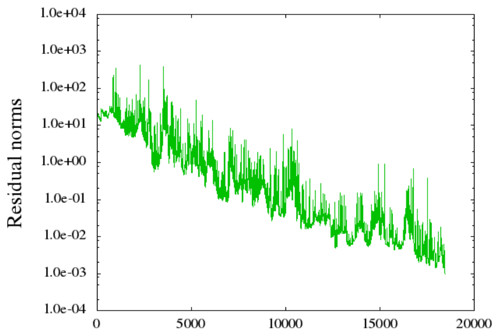
	elements	nodes	subdomains
Mesh(1)	44,314,422	59,051,135	384×1,160
Mesh(2)	8,788,303	11,798,363	80×1,100
Mesh(3)	2,772,796,480	3,704,465,745	11,520×2,432



Iteration counts on the interface
Fig. 2 Convergence histories of Mesh(1)

TABLE II
NUMERICAL RESULTS (MESH(2))

	DOF	Iterations	Time [s]	Memory [MB]
previous	10,346,450	2,173	1,328	424
new	11,059,966	2,186	1,225	574



Iteration counts on the interface
Fig. 3 Convergence histories of Mesh(3)

TABLE III
NUMERICAL RESULTS (MESH(2))

	DOF	Iterations	Time [s]	Memory [MB]
new	3,469,227,540	18,451	31,885	1,306

B. Comparison with the previous method

First, characteristics of convergence of the interface problem are compared with Mesh(1) that diverged in Fig. 1 in the previous method. Computations are performed by Oakleaf-fx [9] with 24 nodes. Fig. 2 shows the convergence histories. The residual norm of the previous method increased from the beginning and diverged finally. On the other hand, the residual norm of the new method reduced smoothly. The characteristic of convergence of the new method has been much improved.

Next, computation times and amounts of memory are compared with Mesh(2) that is able to be solved by the previous method. Computations are performed by a PC cluster that consists of 20 PCs with Intel Core i7-2600. TABLE II shows the numerical results. Because the iterative method is used as the subdomain solver in the previous method and the direct solver is used as the subdomain solver in the new method, amount of memory of the new method increases about 35%. On the other hand, because we can reuse the result of the *LU* decomposition, the computation time of the new method reduced about 8 %.

C. Analysis of the model with 3.5 billion DOF

To show effectiveness of the new method for analysis of the model with over 1 billion DOF, Mesh(3) is computed by Oakleaf-fx with 720 nodes. Fig. 3 shows the convergence history and TABLE III shows the numerical result. We have succeeded to solve the time-harmonic eddy current problem with 3.5 billion DOF in about 9 hours.

IV. CONCLUSIONS

To improve the characteristic of convergence of the interface problem in the time-harmonic eddy current problem, the HDDM has been applied to the mixed formulation with the Lagrange multiplier that is not singular in the *A* method. As a result, the characteristic of convergence of the new method has been much improved. Furthermore, the model for the accuracy verification of the eddy current analysis and uses the solenoidal coil with unlimited length with 3.5 billion DOF has been solved in about 9 hours.

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